

CAD Using New Algorithms for nth Order Butterworth Digital High-Pass Filter

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Abstract This paper gives the description of new algorithms for constructing the transfer function of nth order Butterworth digital HPF by the use of the fact of multiplication of cascade transfer functions of second order and first order digital HPFs. By using computer-aided design with new algorithms for calculation of z-domain, transfer function for (conjugate poles and single real pole) can be achieved. To perform above requirements, linear programming is used to create C++ program for the collection of cascade combination. The algorithms are very exact, flexible, and fast. Users of this program can make design of different sampling frequencies, cut-off frequencies, and any order of Butterworth digital HPF with very high flexibility.

Keywords: Programming algorithms, Digital filter, IIR filter.

1. INTRODUCTION

This paper shows new algorithms of design of Butterworth digital HPF. By using these algorithms, it is very easy and flexible to get any order of Butterworth digital HPF which can be used inside printed digital circuits or integrated circuits. This design is fast and suitable to be used with adaptive digital systems and real time digital systems.

The detection of wanted signals is the mission of desired filtering. By filtering, some signals are allowed to pass, while the others are prevented. Butterworth filters have a smooth pass-band without ripples, which makes them very desirable for filtering[1].

Digital filtering can be considered as one of the most powerful tools of DSP. Digital filtering is desired in adaptive systems because its operation is determined by a program stored in the processor's memory. Which means Digital filtering can be easily changed without changing the hardware[2].

Thede [3] has written software in C Language for IIR filter design. He has made the conversion of transfer functions from s-domain to z-domain using bilinear transformation.

Kumar[4] has made CAD techniques for FIR and IIR digital filters.

Chauhan[5] has suggested algorithms for the design of IIR digital filter.

2. TRANSFER FUNCTION OF BUTTERWORTH DIGITAL HPF

Butterworth HPFs are designed to have amplitude response characteristic that has relatively very smooth pass-band without ripples.

By approximating the ideal Butterworth high-pass filter which has a relatively flat pass-band characteristic, we get [6]:

$$|H(jw)| = \frac{k(j\frac{w}{w_c})^n}{\sqrt{1+(\frac{w}{w_c})^{2n}}} \dots\dots\dots (1)$$

That w represents the angular frequency in rad/sec.

n represents the order of the filter.

w_c represents the cutoff-frequency of the high-pass filter in rad/sec.

k is constant.

Calculating the poles of nth order Butterworth HPF can be by [6]:

$$P_m = -w_c \sin \frac{(2m-1)\pi}{2n} + jw_c \cos \frac{(2m-1)\pi}{2n} \dots (2)$$

That m=1, 2, ... , n

Then using above equation all poles can be calculated.

3. FIRST ORDER BUTTERWORTH DIGITAL HPF

The first order transfer function of Butterworth HPF is :

$$H(s) = \frac{s}{s + w_c} \dots (3)$$

Where w_c is the cut-off frequency in rad./sec.

Using division, equation (3) will be:

$$H(s) = 1 - \frac{w_c}{s + w_c} \dots (4)$$

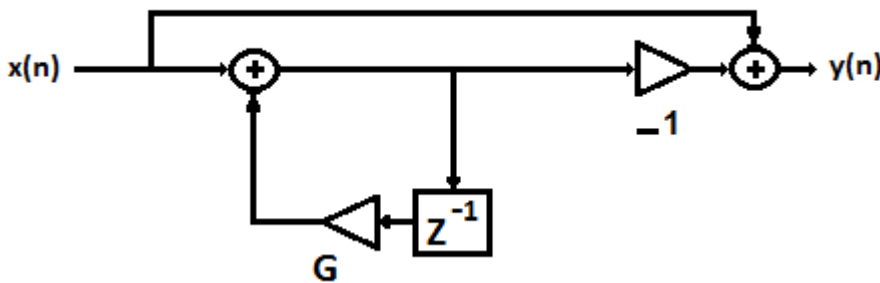
Using the relations between Laplace Domain and Z Domain, Equation (4) will be:

$$H(z) = 1 + \frac{-1}{1 - e^{-w_c T} z^{-1}} \dots (5)$$

That T represents the sampling period. Let

$$G = e^{-w_c T} \dots (6)$$

Then above transfer function of Equation (5) can be represented by following IIR digital circuit figure:



Figure(1): First Order Butterworth Digital HPF.

4. SECOND ORDER BUTTERWORTH DIGITAL HPF

The second order transfer function of Butterworth high-pass filter is:

$$H(s) = \frac{s^2}{(s+b+jc)(s+b-jc)} \dots (7)$$

Where $-b - jc$, $-b + jc$ are conjugate poles calculated from Equation (2) .

That Butterworth transfer functions consist of conjugate poles if the order is even and additional single real pole if the order of transfer function is odd. From Equation(5), we get:

$$H(s) = \frac{s^2}{(s^2+b)^2+c^2} \dots\dots\dots (8)$$

By division:

$$H(s) = 1 + \frac{-2bs-b^2-c^2}{(s^2+b)^2+c^2} \dots\dots\dots (9)$$

$$\Rightarrow H(s) = 1 - (2b) \cdot \frac{(s+b)}{(s^2+b)^2+c^2} - \left(\frac{b^2-c^2}{c}\right) \cdot \frac{c}{(s^2+b)^2+c^2} \dots\dots\dots (10)$$

Using the relations between Laplace Domain and Z Domain, we get:

$$H(z) = 1 - (2b) \cdot \frac{1-e^{-bT}\cos(cT)z^{-1}}{1-2e^{-bT}\cos(cT)z^{-1}+e^{-2bT}z^{-2}} - \left(\frac{b^2-c^2}{c}\right) \cdot \frac{e^{-bT}\sin(cT)z^{-1}}{1-2e^{-bT}\cos(cT)z^{-1}+e^{-2bT}z^{-2}} \dots\dots (11)$$

That T is the sampling period.

$$\text{Now let } K1 = -2b \dots\dots\dots (12)$$

$$K2 = 2be^{-bT}\cos(cT) \dots\dots\dots (13)$$

$$K3 = \frac{b^2-c^2}{c} e^{-bT} \sin(cT) \dots\dots\dots (14)$$

$$B1 = -2e^{-bT}\cos(cT) \dots\dots\dots (15)$$

$$B2 = e^{-2bT} \dots\dots\dots (16)$$

From Equations (11), (12), (13), (14), (15), (and (16), we get:

$$H(z) = 1 + \frac{K1-K2 z^{-1}}{1+B1 z^{-1}+B2 z^{-2}} + \frac{K3 z^{-1}}{1+B1 z^{-1}+B2 z^{-2}} \dots\dots\dots (17)$$

And from Equation (17), we get:

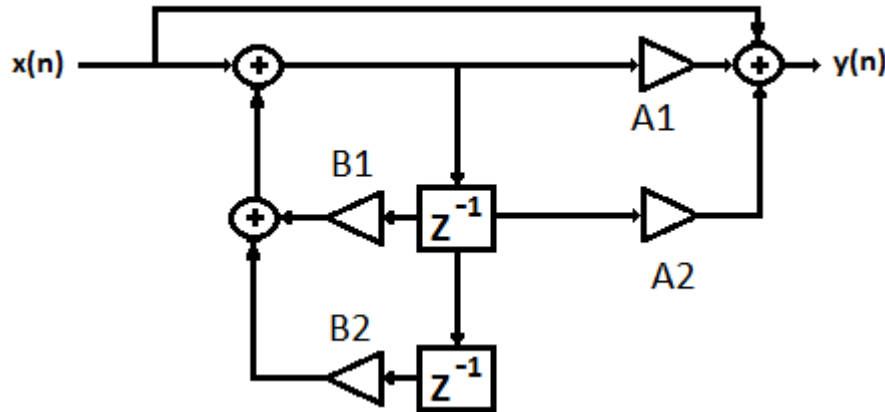
$$H(z) = 1 + \frac{A1 - A2 z^{-1}}{1 + B1 z^{-1} + B2 z^{-2}} \dots\dots\dots (18)$$

Where:

$$A1 = K1 + K3 \dots\dots\dots (19)$$

$$, \text{and } A2 = K2 \dots\dots\dots (20)$$

Then above transfer function of Equation (18) can be represented by following IIR digital circuit figure:



Figure(2): Second Order Butterworth Digital HPF.

5. DESIGN CONCEPTS

Using equation (2) of Butterworth poles generation it is clear that any order can be represented by combination of conjugate poles if the order is even and additional real pole if the order is odd. To represent the transfer function, we can make each two conjugate poles represent a second order transfer function and by the multiplication of these transfer functions we can get the overall transfer function of given order. Actually an additional transfer function which represent the single real pole (if the order is odd) must be given with the combination.

Using circuit representation of above combination, each transfer function of two conjugate poles can be represented by a second order circuit while the single real pole can be represented by a single first order circuit. The multiplication of above transfer functions will represent the overall transfer function of filter. The multiplication is cascade circuits representation.

For example if the designed filter is 11th order then the cascade second order stages and single first order stage will be as shown in Figure (3).

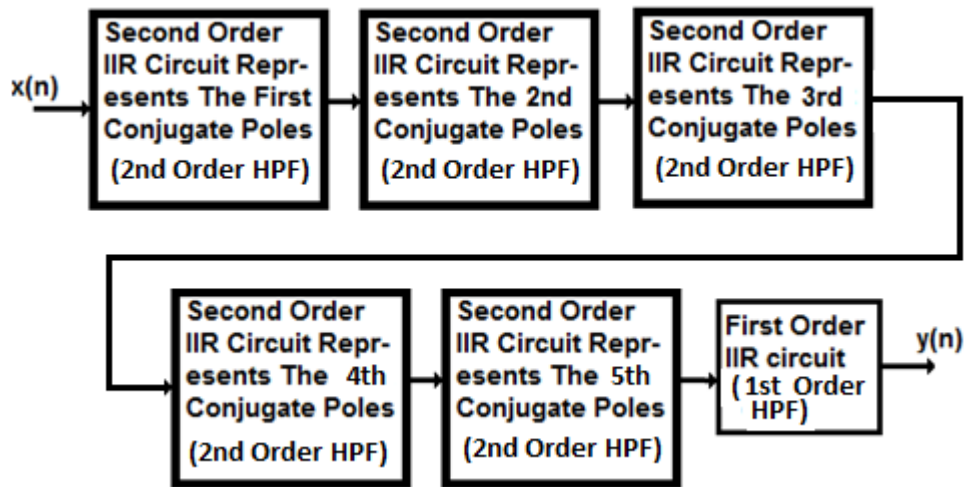


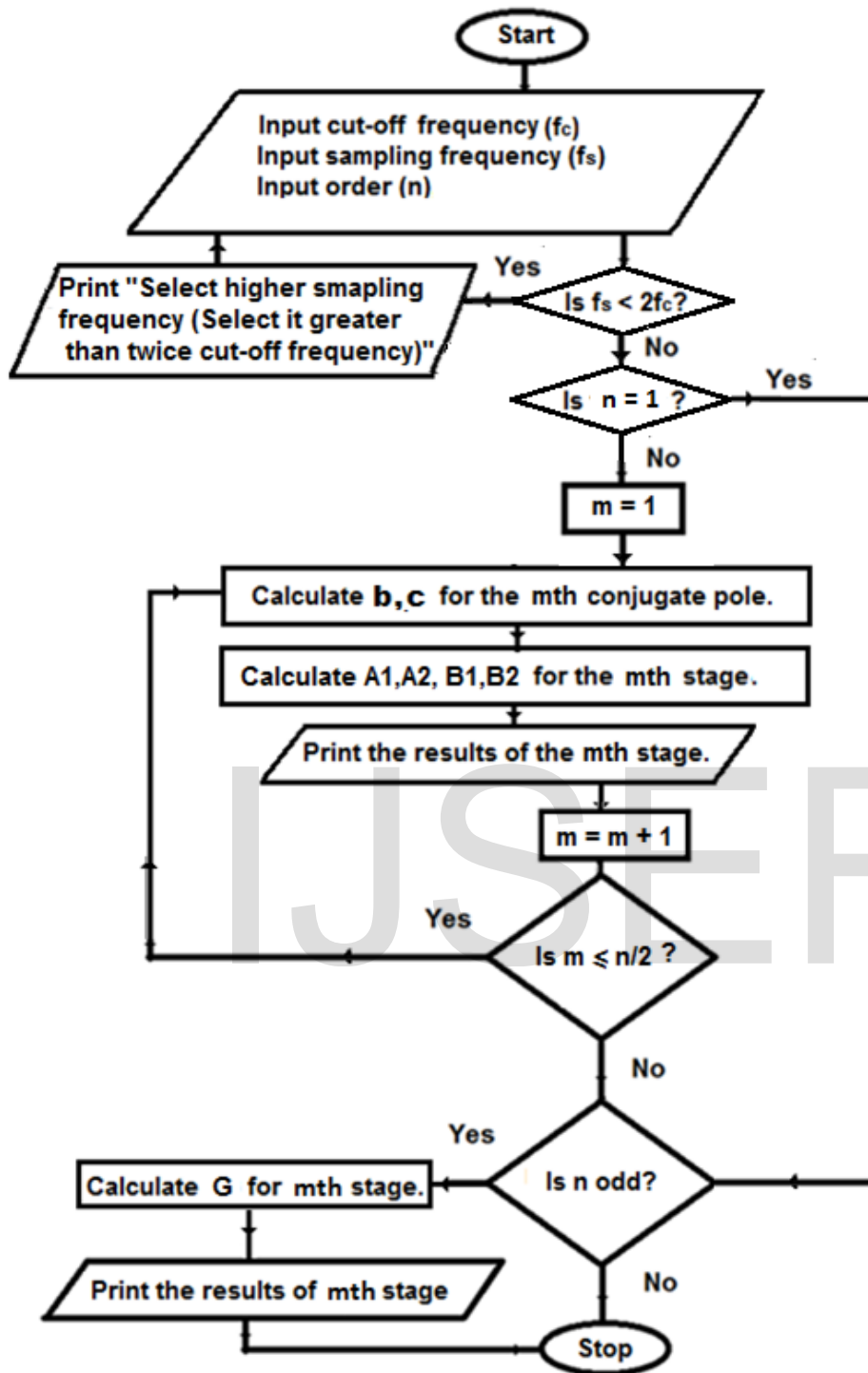
Figure (3): Stages of 11th order Butterworth Digital HPF.

To calculate the coefficients of each stage, we must depend on the calculated conjugate poles by using equations (15), (16), (19) and (20).

6. DESIGN ALGORITHMS

Flowchart (1) shows the algorithms of n th order Butterworth digital HPF which gives high flexibility to choose (sampling, cut-off) frequencies, and the order of the Butterworth digital HPF.

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Flowchat(1): The algorithms of nth order Butterworth digital HPF design program.

7. EXAMPLES OF DESIGN

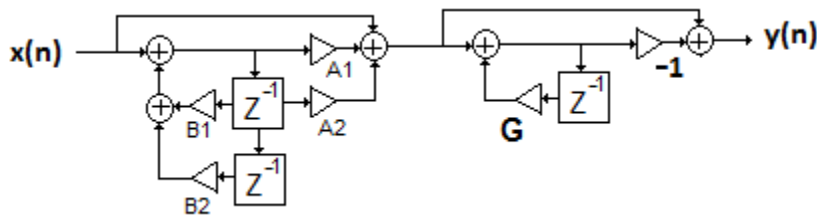
Examples of design of the program (written in C++ Language) which follows the algorithms of Flowchart(1) are given as follows:

- For third order with sampling freq.=166kHz and with selected cut-off frequency $f_c=46\text{kHz}$.

- The first stage: $A1=-1.695269$, $A2=0.045855$, $B1=-0.052672$, $B2=0.175323$

- The second stage : a single real pole stage of $G=0.175323$

Fig.(4) shows above third order Butterworth Digital HPF.

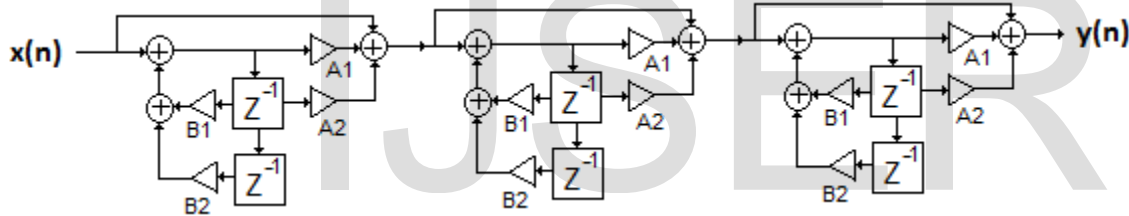


Figure(4): Third Order Butterworth Digital HPF

- For sixth order with sampling freq.=166kHz and with selected cut-off frequency $f_c=46$ kHz.

- The first stage: $A1=-0.964890$, $A2=-0.63618$, $B1=0.141173$, $B2=0.406053$
- The second stage: $A1=-2.222829$, $A2=0.239492$, $B1=-0.194525$, $B2=0.085237$
- The third stage: $A1=-2.800301$, $A2=0.563292$, $B1=-0.334934$, $B2=0.034611$

Fig.(5) shows above sixth order Butterworth Digital HPF.

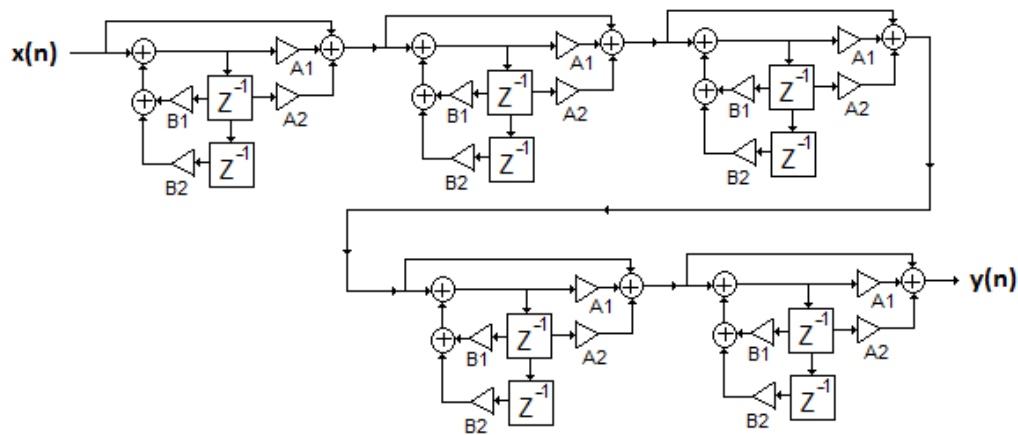


Figure(5): Sixth Order Butterworth Digital HPF

- For tenth order with sampling freq.=166kHz and with selected cut-off frequency $f_c=46$ kHz.

- The first stage: $A1=-0.606285$, $A2=-0.061541$, $B1=0.225945$, $B2=0.579991$
- The second stage: $A1=-1.566964$, $A2=0.013943$, $B1=-0.017640$, $B2=0.205788$
- The third stage: $A1=-2.222829$, $A2=0.239492$, $B1=-0.194525$, $B2=0.085237$
- The fourth stage: $A1=-2.640031$, $A2=0.462674$, $B1=-0.298239$, $B2=0.044928$
- The fifth stage: $A1=-2.846018$, $A2=0.593357$, $B1=-0.345038$, $B2=0.032085$

Fig.(6) shows above tenth order Butterworth Digital HPF.



Figure(6): Tenth Order Butterworth Digital HPF

8. CONCLUSIONS

The designed program for Butterworth digital HPF, is very simple to be used and does not need knowledge in electronic filtering design in order to run it. The performance of all designed filters have been evaluated using "Electronic Workbench 10", that the design of this CAD program is exact and convenient for any practical use.

The outputs of this program are the coefficients of each stage of the Butterworth digital HPF.

REFERENCES

- [1] Rabin Raut and M. N. S. Swamy, "Modern Analog Filter Analysis and Design", WILEY-VCH Verlag & Co. KGaA, 2010.
- [2] Wai-Kai Chen, "Passive, Active, and Digital Filters (The Circuits and Filters Handbook, 3rd Edition)", Taylor and Francis Group, 2009.
- [3] Les Thede, "Practical Analog and Digital Filter Design", Artech House, Inc., 2004.
- [4] A. Anannd Kumar, "Digital Signal Processing", PHI Learning Private Limited, 2015.
- [5] R.S. Chauhan and Sandeep K. Arya, "Design of IIR digital filter using analog to digital mapping", Journal of Neural Computing Systems, Vol. 03, No. 01, 2010.
- [6] Andreas Antoniou, "Digital Filters: Analysis, Design, and Signal Processing Applications", McGraw Hill Professional, 2018.
- [7] Jose Maria Giron-Sierra, "Digital Signal Processing with Matlab Examples", Springer, 2017.
- [8] Li Tan and Jean Jiang, "Digital Signal Processing: Fundamentals and Applications", Elsevier Inc., 2013.
- [9] B. Preetham Kumar, "Digital Signal Processing Laboratory", Taylor and Francis Group, 2010.
- [10] J. G. Proakis and D. G. Manolakis, "Digital Signal Processing: Principles, Algorithms, and Applications", Pearson Education Ltd., 2007.
- [11] R.S. Chauhan and Sandeep K. Arya, "Design of IIR digital filter using analog to digital mapping", Journal of Neural Computing Systems. 3(Jan. 2010), 51-55., 2010.
- [12] Jiang A, Kwan HK, "Minimax design of IIR digital filters using iterative SOCP". IEEE Trans Circuits Syst I Reg Papers 57(6):1326-1337, 2010.
- [13] Sande Seema Bhogeshwar, M.K.Soni, Dipail Bansal, "Design of Simulink Model to denoise ECG Signal Using Various IIR &

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